TRIANGULAR WAVE FUNCTION: PROBABILITIES

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 1.4.

The square modulus of the wave function which is the solution to the Schrödinger equation is interpreted as a probability density. As an example consider the wave function given by

$$\Psi(x,0)=\begin{cases} A\frac{x}{a} & 0\le x\le a\\ A\frac{b-x}{b-a} & a\le x\le b\\ 0 & \text{otherwise}\end{cases} \tag{1}$$

We can normalize Ψ by requiring

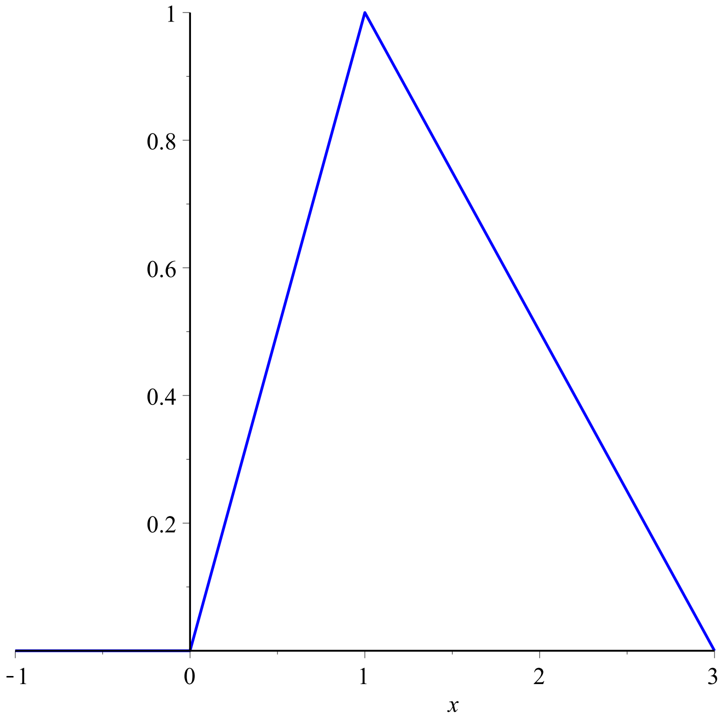
$$\int_0^b |\Psi|^2\,dx = 1 \tag{2}$$

Plugging in the formula and doing the integral gives

$$\begin{aligned}\int_0^b |\Psi|^2\,dx &= |A|^2\left[\int_0^a \frac{x^2}{a^2}dx+\int_a^b\left(\frac{b-x}{b-a}\right)^2 dx\right] &(3)\\ &= |A|^2\frac{b}{3} &(4)\\ A &= \sqrt{\frac{3}{b}} &(5)\end{aligned}$$

where we've taken the positive real root for A. Note that A could also be multiplied by a phase factor $e^{i\delta}$ for any real δ without affecting normalization. This can be important in some applications where we need to add together wave functions.

Given this value for A, we can plot 1. Here, we've taken $a=1$ and $b=3$:



Since Ψ has its maximum at $x=a$, that is where the particle is most likely to be found. The probability of the particle being found to the left of $x=a$ is

$$P_{x<a}=\frac{3}{b}\int_0^a\frac{x^2}{a^2}dx=\frac{a}{b} \tag{6}$$

If $b=a$, then Ψ drops to zero at $x=a$ so $P_{x<a}=1$. If $b=2a$, then Ψ is an isosceles triangle symmetric about $x=a$ so $P_{x<a}=\frac{1}{2}$.

The expectation value of x is

$$\langle x\rangle=\int x\,|\Psi|^2\,dx=\frac{a}{2}+\frac{b}{4} \tag{7}$$

where we used Maple to simplify the integration. If $b=2a$, then $\langle x\rangle=a$ as expected.