DELTA FUNCTION WELL: STATISTICS

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 1.5.

The delta function well gives rise to a wave function that decays exponentially either side of the delta function:

$$\Psi(x,t) = Ae^{-\lambda|x|}e^{-i\omega t} \tag{0.1}$$

We can normalize Ψ in the usual way:

$$\int_{-\infty}^{\infty}|\Psi|^2\,dx = |A|^2\left[\int_{-\infty}^{0}e^{2\lambda x}dx + \int_{0}^{\infty}e^{-2\lambda x}dx\right] \tag{0.2}$$

$$= 2|A|^2\int_0^\infty e^{-2\lambda x}dx \tag{0.3}$$

$$= \frac{|A|^2}{\lambda} \tag{0.4}$$

$$A = \sqrt{\lambda} \tag{0.5}$$

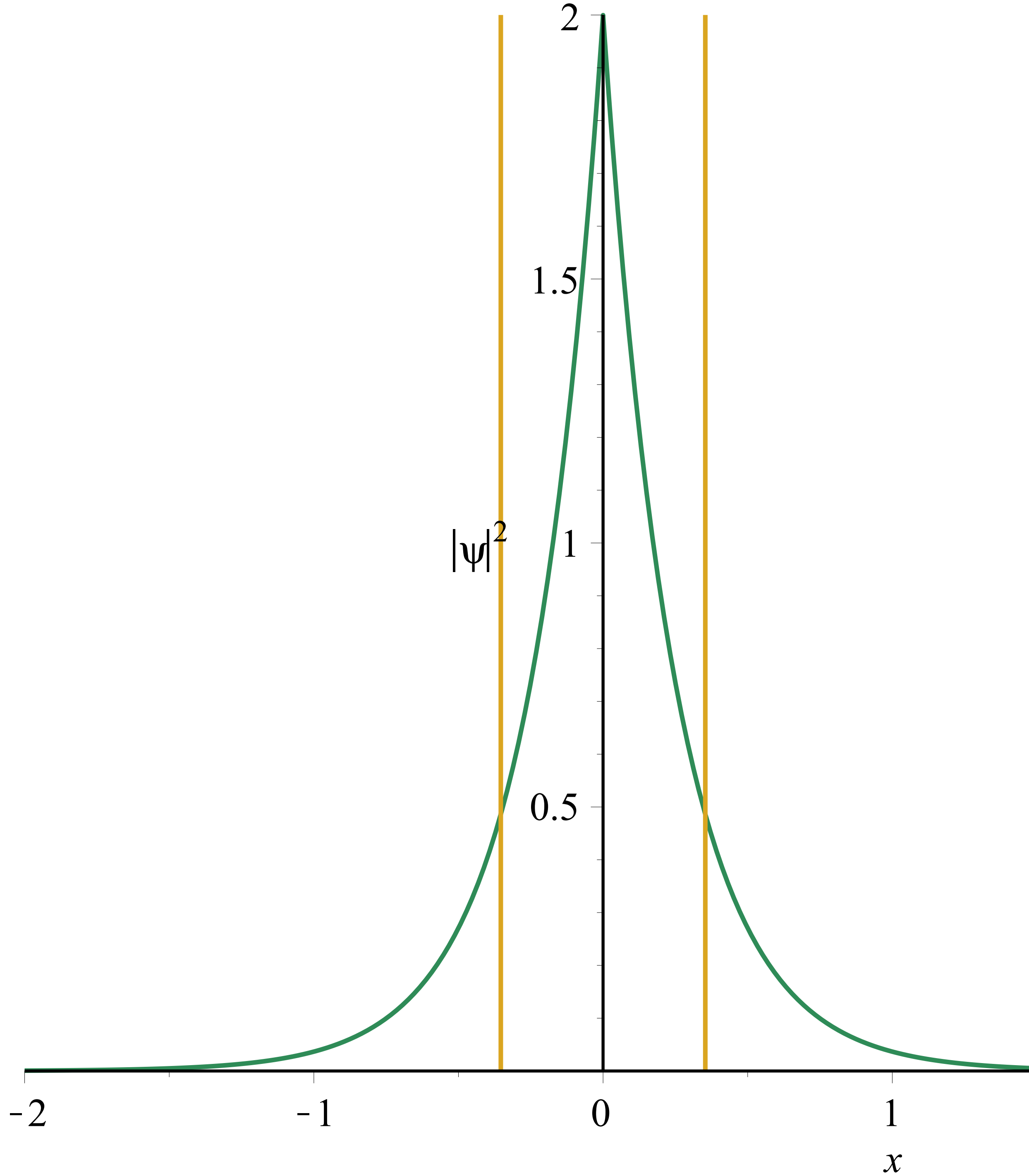
By symmetry, $\langle x\rangle = 0$ and

$$\left\langle x^2\right\rangle = 2\lambda\int_0^\infty x^2 e^{-2\lambda x}dx = \frac{1}{2\lambda^2} \tag{0.6}$$

Therefore

$$\sigma = \sqrt{\langle x^2\rangle - \langle x\rangle^2} = \frac{1}{\sqrt{2}\lambda} \tag{0.7}$$

A plot of $|\Psi|^2$ is shown, with vertical yellow lines indicating $x = \pm\frac{1}{\sqrt{2}\lambda}$, for the case $\lambda = 2$:



The probability that the particle lies outside $x = \pm\frac{1}{\sqrt{2}\lambda}$ is

$$P_{|x|>\sigma} = 2\lambda \int_{1/\sqrt{2}\lambda}^{\infty} e^{-2\lambda x} dx = \frac{1}{e^{\sqrt{2}}} = 0.2431 \tag{0.8}$$

In this case, the probability of x being greater than one standard deviation is a constant, independent of λ.