DELTA FUNCTION WELL: STATISTICS

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Post date: 23 Jun 2015.

Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 1.5.

The delta function well gives rise to a wave function that decays exponentially either side of the delta function:

$$\Psi(x,t) = Ae^{-\lambda|x|}e^{-i\omega t} \tag{1}$$

We can normalize Ψ in the usual way:

$$\begin{aligned}
\int_{-\infty}^{\infty}|\Psi|^2 dx &= |A|^2\left[\int_{-\infty}^{0}e^{2\lambda x}dx+\int_{0}^{\infty}e^{-2\lambda x}dx\right] && (2)\\
&= 2|A|^2\int_0^{\infty}e^{-2\lambda x}dx && (3)\\
&= \frac{|A|^2}{\lambda} && (4)\\
A &= \sqrt{\lambda} && (5)
\end{aligned}$$

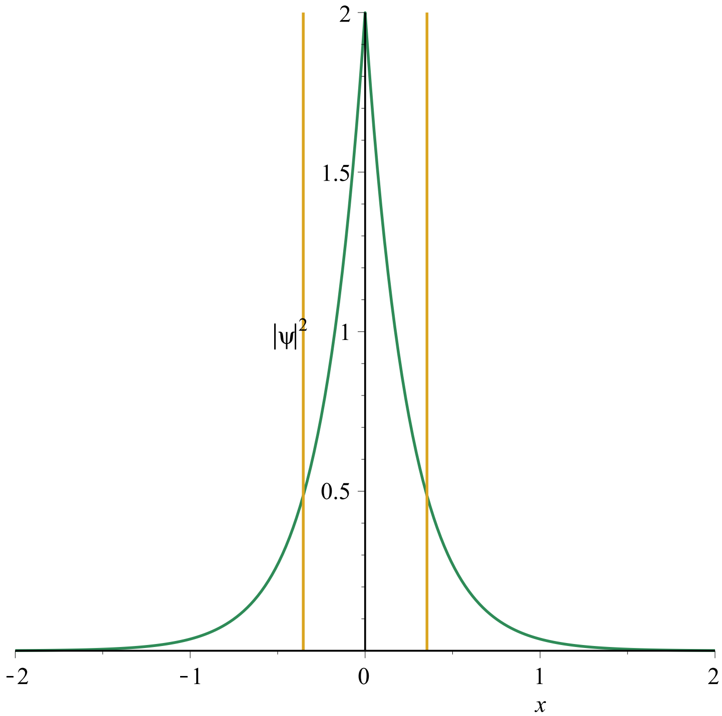
By symmetry, $\langle x\rangle = 0$ and

$$\left\langle x^2\right\rangle = 2\lambda\int_0^{\infty}x^2e^{-2\lambda x}dx = \frac{1}{2\lambda^2} \tag{6}$$

Therefore

$$\sigma = \sqrt{\langle x^2\rangle - \langle x\rangle^2} = \frac{1}{\sqrt{2}\lambda} \tag{7}$$

A plot of $|\Psi|^2$ is shown, with vertical yellow lines indicating $x = \pm\frac{1}{\sqrt{2}\lambda}$, for the case $\lambda = 2$:



The probability that the particle lies outside $x=\pm\frac{1}{\sqrt{2}\lambda}$ is

$$P_{|x|>\sigma}=2\lambda\int_{1/\sqrt{2}\lambda}^{\infty}e^{-2\lambda x}dx=\frac{1}{e^{\sqrt{2}}}=0.2431 \tag{8}$$

In this case, the probability of x being greater than one standard deviation is a constant, independent of λ.