

ADDING A CONSTANT TO THE POTENTIAL INTRODUCES A PHASE FACTOR

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 1.8.

The time-independent Schrödinger equation in one dimension can be separated into two equations as follows:

$$(1) \quad -\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

$$(2) \quad i\hbar \frac{d\Xi(t)}{dt} = E\Xi(t)$$

and the general solution is

$$(3) \quad \Psi(x,t) = \psi(x)\Xi(t)$$

The time component can be solved as

$$(4) \quad \Xi(t) = Ce^{-iEt/\hbar}$$

where C is the constant of integration.

If we add a constant (in both space and time) V_0 to the potential, then the original Schrödinger equation becomes

$$(5) \quad -\frac{\hbar^2}{2m} \frac{d^2\Psi}{dx^2} + V(x)\Psi + V_0\Psi = i\hbar \frac{\partial\Psi}{\partial t}$$

$$(6) \quad -\frac{\hbar^2}{2m} \frac{d^2\Psi}{dx^2} + V(x)\Psi = i\hbar \frac{\partial\Psi}{\partial t} - V_0\Psi$$

Applying separation of variables gives us

$$(7) \quad -\frac{\hbar^2}{2m} \frac{1}{\psi(x)} \frac{\partial^2\psi(x)}{\partial x^2} + V(x) = E$$

$$(8) \quad i\hbar \frac{1}{\Xi(t)} \frac{\partial\Xi}{\partial t} - V_0 = E$$

[Since V_0 is independent of both x and t , we could put it in either the $\psi(x)$ or the $\Xi(t)$ equation, but putting it in the Ξ equation eliminates it from the more complex ψ equation, so we'll do that.]

The solution to 8 is now

$$(9) \quad \Xi(t) = Ce^{-i(E+V_0)t/\hbar}$$

so we've introduced a phase factor $e^{-iV_0t/\hbar}$ into the overall wave function Ψ . For the time-independent Schrödinger equation, all quantities of physical interest involve multiplying the complex conjugate Ψ^* by some operator $\hat{Q}(x)$ that depends only on x , operating on Ψ . That is, we're interested only in quantities of the form

$$(10) \quad \Psi^* [\hat{Q}(x) \Psi] = |C|^2 e^{+i(E+V_0)t/\hbar} e^{-i(E+V_0)t/\hbar} \psi^* [\hat{Q}(x) \psi]$$

$$(11) \quad = |C|^2 \psi^* [\hat{Q}(x) \psi]$$

Thus the phase factor disappears when calculating any physical quantity.