

## ADDING A CONSTANT TO THE POTENTIAL INTRODUCES A PHASE FACTOR

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 1.8.

The time-independent Schrödinger equation in one dimension can be separated into two equations as follows:

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x) \quad (1)$$

$$i\hbar \frac{d\Xi(t)}{dt} = E\Xi(t) \quad (2)$$

and the general solution is

$$\Psi(x,t) = \psi(x)\Xi(t) \quad (3)$$

The time component can be solved as

$$\Xi(t) = Ce^{-iEt/\hbar} \quad (4)$$

where  $C$  is the constant of integration.

If we add a constant (in both space and time)  $V_0$  to the potential, then the original Schrödinger equation becomes

$$-\frac{\hbar^2}{2m} \frac{d^2\Psi}{dx^2} + V(x)\Psi + V_0\Psi = i\hbar \frac{\partial\Psi}{\partial t} \quad (5)$$

$$-\frac{\hbar^2}{2m} \frac{d^2\Psi}{dx^2} + V(x)\Psi = i\hbar \frac{\partial\Psi}{\partial t} - V_0\Psi \quad (6)$$

Applying separation of variables gives us

$$-\frac{\hbar^2}{2m} \frac{1}{\psi(x)} \frac{\partial^2\psi(x)}{\partial x^2} + V(x) = E \quad (7)$$

$$i\hbar \frac{1}{\Xi(t)} \frac{\partial\Xi}{\partial t} - V_0 = E \quad (8)$$

[Since  $V_0$  is independent of both  $x$  and  $t$ , we could put it in either the  $\psi(x)$  or the  $\Xi(t)$  equation, but putting it in the  $\Xi$  equation eliminates it from the more complex  $\psi$  equation, so we'll do that.]

The solution to 8 is now

$$\Xi(t) = C e^{-i(E+V_0)t/\hbar} \quad (9)$$

so we've introduced a phase factor  $e^{-iV_0t/\hbar}$  into the overall wave function  $\Psi$ . For the time-independent Schrödinger equation, all quantities of physical interest involve multiplying the complex conjugate  $\Psi^*$  by some operator  $\hat{Q}(x)$  that depends only on  $x$ , operating on  $\Psi$ . That is, we're interested only in quantities of the form

$$\Psi^* [\hat{Q}(x) \Psi] = |C|^2 e^{+i(E+V_0)t/\hbar} e^{-i(E+V_0)t/\hbar} \psi^* [\hat{Q}(x) \psi] \quad (10)$$

$$= |C|^2 \psi^* [\hat{Q}(x) \psi] \quad (11)$$

Thus the phase factor disappears when calculating any physical quantity.