DELTA FUNCTION - FOURIER TRANSFORM

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.26.

The Dirac delta function is defined by the two conditions

$$\delta(x) = 0 \text{ if } x \neq 0 \tag{1}$$

$$\int_{-\infty}^{\infty} \delta(x)dx = 1 \tag{2}$$

Using Plancherel's theorem, we can find the Fourier transform $\Delta(k)$ of $\delta(x)$:

$$\Delta(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \delta(x) e^{-ikx} dx \tag{3}$$

$$= \frac{1}{\sqrt{2\pi}} \tag{4}$$

The inverse of this relation is

$$\delta(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \Delta(k) e^{ikx} dk \tag{5}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{ikx} dk \tag{6}$$

This formula is clearly nonsense, since we're integrating an oscillating function over an infinite range, so it doesn't converge. Griffiths states that the formula "can be extremely useful, if handled with care." I'm not entirely convinced by this; a dodgy formula is still a dodgy formula, no matter how carefully you handle it. Nevertheless, it's used in a lot of quantum mechanics and the results do seem to be verified by experiment, so I guess we'll have to accept it.

[A derivation of $\delta(x)$ that makes this result look a bit more believeable is given here.]

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