

ROTATION MATRICES - MATRIX ELEMENTS

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References: Shankar, R. (1994), *Principles of Quantum Mechanics*, Plenum Press. Exercise 1.6.1.

We can represent a linear operator Ω by its matrix representation in a given basis. If the basis e_i is orthonormal then the matrix elements are given by

$$\Omega_{ij} = \langle e_i | \Omega | e_j \rangle \quad (1)$$

The rotation operator $R\left(\frac{\pi}{2}\mathbf{i}\right)$ that rotates vectors by $\frac{\pi}{2}$ about the x axis can be written in terms of the orthonormal basis consisting of the unit vectors $|1\rangle, |2\rangle, |3\rangle$ along the three coordinate axes by examining the effect that it has on each vector in the basis. It leaves $|1\rangle$ unchanged, rotates $|2\rangle$ into $|3\rangle$, and $|3\rangle$ into $-|2\rangle$ so we have

$$R\left(\frac{\pi}{2}\mathbf{i}\right) |1\rangle = |1\rangle \quad (2)$$

$$R\left(\frac{\pi}{2}\mathbf{i}\right) |2\rangle = |3\rangle \quad (3)$$

$$R\left(\frac{\pi}{2}\mathbf{i}\right) |3\rangle = -|2\rangle \quad (4)$$

We can work out the matrix elements by applying 1 to these three transformation equations:

$$R\left(\frac{\pi}{2}\mathbf{i}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \quad (5)$$

As another example, suppose we have the matrix (also in the same basis)

$$\Omega = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (6)$$

This has the effects

$$\Omega|1\rangle = |2\rangle \quad (7)$$

$$\Omega|2\rangle = |3\rangle \quad (8)$$

$$\Omega|3\rangle = |1\rangle \quad (9)$$

Thus Ω cyclically permutes the three basis vectors, which is equivalent to a rotation by $\frac{2\pi}{3}$ about the line $x = y = z$.

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