HERMITIAN OPERATORS - A FEW EXAMPLES

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References: Shankar, R. (1994), *Principles of Quantum Mechanics*, Plenum Press. Exercise 1.6.2.

Here are a few more results about hermitian operators.

Suppose we are given two hermitian operators Ω and Λ. We'll look at some combinations of these operators.

The operator $\Omega\Lambda$ has the hermitian conjugate

$$(\Omega\Lambda)^{\dagger}=\Lambda^{\dagger}\Omega^{\dagger}=\Lambda\Omega \tag{1}$$

Thus the product operator $\Omega\Lambda$ is hermitian only if Λ and Ω commute.

The operator $\Omega\Lambda+\lambda\Omega$ for some complex scalar λ has the hermitian conjugate

$$(\Omega\Lambda+\lambda\Omega)^{\dagger}=\Lambda^{\dagger}\Omega^{\dagger}+\lambda^{*}\Omega^{\dagger} \tag{2}$$

$$=\Lambda\Omega+\lambda^{*}\Omega \tag{3}$$

This operator is therefore hermitian only if Λ and Ω commute and λ is real.

The commutator has the hermitian conjugate

$$[\Omega,\Lambda]^{\dagger}=(\Omega\Lambda-\Lambda\Omega)^{\dagger} \tag{4}$$

$$=\Lambda\Omega-\Omega\Lambda \tag{5}$$

$$=[\Lambda,\Omega] \tag{6}$$

$$=-[\Omega,\Lambda] \tag{7}$$

Thus the commutator is anti-hermitian (the hermitian conjugate is the negative of the original operator).

Finally, what happens if we multiply the commutator by i?

$$(i[\Omega,\Lambda])^{\dagger}=-i(\Omega\Lambda-\Lambda\Omega)^{\dagger} \tag{8}$$

$$=-i(\Lambda\Omega-\Omega\Lambda) \tag{9}$$

$$=-i[\Lambda,\Omega] \tag{10}$$

$$=i[\Omega,\Lambda] \tag{11}$$

Thus the operator $i\left[\Omega,\Lambda\right]$ is hermitian.