HERMITIAN OPERATORS - A FEW EXAMPLES

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References: Shankar, R. (1994), *Principles of Quantum Mechanics*, Plenum Press. Exercise 1.6.2.

Here are a few more results about hermitian operators.

Suppose we are given two hermitian operators Ω and Λ. We'll look at some combinations of these operators.

The operator $\Omega\Lambda$ has the hermitian conjugate

$$(\Omega\Lambda)^{\dagger}=\Lambda^{\dagger}\Omega^{\dagger}=\Lambda\Omega \tag{1}$$

Thus the product operator $\Omega\Lambda$ is hermitian only if Λ and Ω commute.

The operator $\Omega\Lambda+\lambda\Omega$ for some complex scalar λ has the hermitian conjugate

$$\begin{aligned}(\Omega\Lambda+\lambda\Omega)^{\dagger} &= \Lambda^{\dagger}\Omega^{\dagger}+\lambda^{*}\Omega^{\dagger} && (2)\\ &= \Lambda\Omega+\lambda^{*}\Omega && (3)\end{aligned}$$

This operator is therefore hermitian only if Λ and Ω commute and λ is real.

The commutator has the hermitian conjugate

$$\begin{aligned}[\Omega,\Lambda]^{\dagger} &= (\Omega\Lambda-\Lambda\Omega)^{\dagger} && (4)\\ &= \Lambda\Omega-\Omega\Lambda && (5)\\ &= [\Lambda,\Omega] && (6)\\ &= -[\Omega,\Lambda] && (7)\end{aligned}$$

Thus the commutator is anti-hermitian (the hermitian conjugate is the negative of the original operator).

Finally, what happens if we multiply the commutator by i?

$$\begin{aligned}(i[\Omega,\Lambda])^{\dagger} &= -i(\Omega\Lambda-\Lambda\Omega)^{\dagger} && (8)\\ &= -i(\Lambda\Omega-\Omega\Lambda) && (9)\\ &= -i[\Lambda,\Omega] && (10)\\ &= i[\Omega,\Lambda] && (11)\end{aligned}$$

Thus the operator $i[\Omega,\Lambda]$ is hermitian.