

HERMITIAN MATRICES - EXAMPLE WITH 4 MATRICES

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References: Shankar, R. (1994), *Principles of Quantum Mechanics*, Plenum Press. Exercise 1.8.8.

Suppose we have four hermitian matrices M^i for $i = 1, 2, 3, 4$ that obey the relation

$$M^i M^j + M^j M^i = 2\delta^{ij} I \quad (1)$$

We can find the possible eigenvalues as follows. Suppose we choose an orthonormal basis (such a basis always exists for a hermitian matrix) $\{e\}$ in which M^i is diagonal for one particular value of i . That is, for a basis vector $|e_k\rangle$ in this basis, we have $M^i |e_k\rangle = \omega_k^i |e_k\rangle$, where ω_k^i is the k th eigenvalue of M^i .

Then with $i = j$ above, we have

$$2(M^i)^2 = 2I \quad (2)$$

$$(M^i)^2 = I \quad (3)$$

Operating on a vector e from this basis, we get

$$(M^i)^2 |e_k\rangle = |e_k\rangle \quad (4)$$

$$= (\omega_k^i)^2 |e_k\rangle \quad (5)$$

Therefore, the possible values of ω_k^i are ± 1 . We didn't choose any particular value for i , so this is true of all four matrices.

Now, for $i \neq j$ we have

$$M^i M^j = -M^j M^i \quad (6)$$

We can find the trace of M^j as follows. Assuming $i \neq j$

$$\mathrm{Tr}M^j = \mathrm{Tr}(M^i M^i M^j) \quad (7)$$

$$= -\mathrm{Tr}(M^i M^j M^i) \quad (8)$$

$$= -\mathrm{Tr}(M^i M^i M^j) \quad (9)$$

$$= -\mathrm{Tr}M^j \quad (10)$$

In line 1 we used 3, in line 2 we used 6 and in line 3 we used the cyclic property of the trace. Thus $\mathrm{Tr}M^j = -\mathrm{Tr}M^j = 0$.

Since each M^j has zero trace, the trace is the sum of the eigenvalues and the possible eigenvalues are ± 1 , the eigenvalue $+1$ must occur the same number of times as -1 , meaning that each M^j must have an even number of eigenvalues, so the matrices must be even-dimensional.

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