HAMILTONIAN FOR THE ELECTROMAGNETIC FORCE

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References: Shankar, R. (1994), *Principles of Quantum Mechanics*, Plenum Press. Section 2.6.

Here we derive the equations of motion for the electromagnetic force using the Hamiltonian formalism.

The Hamiltonian is given by

$$H(q,p)=\sum_i p_i\dot{q}_i-L(q,\dot{q}) \tag{1}$$

where the velocities $\dot{q}_i$ are expressed in terms of the positions q_i and momenta p_i. The electromagnetic Lagrangian is

$$L=\frac{1}{2}m\mathbf{v}\cdot\mathbf{v}-q\phi+\frac{q}{c}\mathbf{v}\cdot\mathbf{A} \tag{2}$$

where ϕ is the electric potential and $\mathbf{A}$ is the magnetic potential, with $\mathbf{v}$ the velocity of the charge q with mass m. To convert to the Hamiltonian, we need the momentum, defined as

$$p_i=\frac{\partial L}{\partial\dot{q}_i}$$

In this case, the generalized velocity is given by

$$\dot{q}_i=v_i \tag{3}$$

so we have

$$p_i=mv_i+\frac{q}{c}A_i \tag{4}$$

or, in vector notation

$$\mathbf{p}=m\mathbf{v}+\frac{q}{c}\mathbf{A} \tag{5}$$

$$\mathbf{v}=\frac{\mathbf{p}}{m}-\frac{q}{mc}\mathbf{A} \tag{6}$$

The Lagrangian is therefore

$$L=\frac{|\mathbf{p}-q\mathbf{A}/c|^{2}}{2m}-q\phi+\frac{q}{c}\left(\frac{\mathbf{p}}{m}-\frac{q}{mc}\mathbf{A}\right)\cdot\mathbf{A} \tag{7}$$

The first sum in the Hamiltonian is

$$\sum_{i}p_{i}\dot{q}_{i}=\mathbf{p}\cdot\mathbf{v}=\mathbf{p}\cdot\left(\frac{\mathbf{p}}{m}-\frac{q}{mc}\mathbf{A}\right) \tag{8}$$

The Hamiltonian is then

$$H=\mathbf{p}\cdot\left(\frac{\mathbf{p}}{m}-\frac{q}{mc}\mathbf{A}\right)-\frac{|\mathbf{p}-q\mathbf{A}/c|^{2}}{2m}+q\phi-\frac{q}{c}\left(\frac{\mathbf{p}}{m}-\frac{q}{mc}\mathbf{A}\right)\cdot\mathbf{A} \tag{9}$$

$$=\left(\frac{\mathbf{p}}{m}-\frac{q}{mc}\mathbf{A}\right)\left(\mathbf{p}-\frac{q}{c}\mathbf{A}\right)-\frac{|\mathbf{p}-q\mathbf{A}/c|^{2}}{2m}+q\phi \tag{10}$$

$$=\frac{|\mathbf{p}-q\mathbf{A}/c|^{2}}{2m}+q\phi \tag{11}$$

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