

HAMILTONIAN FOR THE ELECTROMAGNETIC FORCE

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References: Shankar, R. (1994), *Principles of Quantum Mechanics*, Plenum Press. Section 2.6.

Here we derive the equations of motion for the electromagnetic force using the Hamiltonian formalism.

The Hamiltonian is given by

$$H(q, p) = \sum_i p_i \dot{q}_i - L(q, \dot{q}) \quad (1)$$

where the velocities \dot{q}_i are expressed in terms of the positions q_i and momenta p_i . The electromagnetic Lagrangian is

$$L = \frac{1}{2} m \mathbf{v} \cdot \mathbf{v} - q\phi + \frac{q}{c} \mathbf{v} \cdot \mathbf{A} \quad (2)$$

where ϕ is the electric potential and \mathbf{A} is the magnetic potential, with \mathbf{v} the velocity of the charge q with mass m . To convert to the Hamiltonian, we need the momentum, defined as

$$p_i = \frac{\partial L}{\partial \dot{q}_i}$$

In this case, the generalized velocity is given by

$$\dot{q}_i = v_i \quad (3)$$

so we have

$$p_i = mv_i + \frac{q}{c} A_i \quad (4)$$

or, in vector notation

$$\mathbf{p} = m\mathbf{v} + \frac{q}{c} \mathbf{A} \quad (5)$$

$$\mathbf{v} = \frac{\mathbf{p}}{m} - \frac{q}{mc} \mathbf{A} \quad (6)$$

The Lagrangian is therefore

$$L = \frac{|\mathbf{p} - q\mathbf{A}/c|^2}{2m} - q\phi + \frac{q}{c} \left(\frac{\mathbf{p}}{m} - \frac{q}{mc} \mathbf{A} \right) \cdot \mathbf{A} \quad (7)$$

The first sum in the Hamiltonian is

$$\sum_i p_i \dot{q}_i = \mathbf{p} \cdot \mathbf{v} = \mathbf{p} \cdot \left(\frac{\mathbf{p}}{m} - \frac{q}{mc} \mathbf{A} \right) \quad (8)$$

The Hamiltonian is then

$$H = \mathbf{p} \cdot \left(\frac{\mathbf{p}}{m} - \frac{q}{mc} \mathbf{A} \right) - \frac{|\mathbf{p} - q\mathbf{A}/c|^2}{2m} + q\phi - \frac{q}{c} \left(\frac{\mathbf{p}}{m} - \frac{q}{mc} \mathbf{A} \right) \cdot \mathbf{A} \quad (9)$$

$$= \left(\frac{\mathbf{p}}{m} - \frac{q}{mc} \mathbf{A} \right) \left(\mathbf{p} - \frac{q}{c} \mathbf{A} \right) - \frac{|\mathbf{p} - q\mathbf{A}/c|^2}{2m} + q\phi \quad (10)$$

$$= \frac{|\mathbf{p} - q\mathbf{A}/c|^2}{2m} + q\phi \quad (11)$$

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