

## INFINITE SQUARE WELL - FORCE TO DECREASE WELL WIDTH

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References: Shankar, R. (1994), *Principles of Quantum Mechanics*, Plenum Press. Section 5.2, Exercise 5.2.4.

One way of comparing the classical and quantum pictures of a particle in an infinite square well is to calculate the force exerted on the walls by the particle. If a particle is in state  $|n\rangle$ , its energy is

$$(1) \quad E_n = \frac{(n\pi\hbar)^2}{2mL^2}$$

If the particle remains in this state as the walls are slowly pushed in, so that  $L$  slowly decreases, then its energy  $E_n$  will increase, meaning that work is done on the system. The force is the change in energy per unit distance, so the force required is

$$(2) \quad F = -\frac{\partial E_n}{\partial L} = \frac{(n\pi\hbar)^2}{mL^3}$$

If we treat the system classically, then a particle with energy  $E_n$  between the walls is effectively a free particle in this region (since the potential  $V = 0$  there), so all its energy is kinetic. That is

$$(3) \quad E_n = \frac{1}{2}mv^2$$

$$(4) \quad v = \sqrt{\frac{2E_n}{m}}$$

$$(5) \quad = \frac{n\pi\hbar}{mL}$$

The classical particle bounces elastically between the two walls, which means its velocity is exactly reversed at each collision. The momentum transfer in such a collision is

$$(6) \quad \Delta p = 2mv = \frac{2n\pi\hbar}{L}$$

The time between successive collisions on the same wall is

$$(7) \quad \Delta t = \frac{2L}{v} = \frac{2mL^2}{n\pi\hbar}$$

Thus the average force exerted on one wall is

$$(8) \quad \bar{F} = \frac{\Delta p}{\Delta t} = \frac{(n\pi\hbar)^2}{mL^3}$$

Comparing with 2, we see that the quantum and classical forces in this case are the same.