

FERMIONS AND BOSONS IN THE INFINITE SQUARE WELL

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Shankar, R. (1994), *Principles of Quantum Mechanics*, Plenum Press. Chapter 10, Exercise 10.3.4.

Suppose we have two identical particles in an infinite square well. The energy levels in a well of width L are

$$(1) \quad E = \frac{(\pi n \hbar)^2}{2mL^2}$$

where $n = 1, 2, 3, \dots$. The corresponding wave functions are given by

$$(2) \quad \psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

If the total energy of the two particles is $\pi^2 \hbar^2 / mL^2$, the only possible configuration is for both particles to be in the ground state $n = 1$. This means the particles must be bosons, so the state vector is

$$(3) \quad |x_1, x_2\rangle = \frac{2}{L} \sin \frac{\pi x_1}{L} \sin \frac{\pi x_2}{L}$$

If the total energy is $5\pi^2 \hbar^2 / 2mL^2$, then one particle is in the state $n = 1$ and the other is in $n = 2$. Since the states are different, the particles can be either bosons or fermions. For bosons, the state vector is

$$(4) \quad |x_1, x_2\rangle = \frac{1}{\sqrt{2}} \left[\frac{2}{L} \sin \frac{\pi x_1}{L} \sin \frac{2\pi x_2}{L} + \frac{2}{L} \sin \frac{2\pi x_1}{L} \sin \frac{\pi x_2}{L} \right]$$

$$(5) \quad = \frac{\sqrt{2}}{L} \left[\sin \frac{\pi x_1}{L} \sin \frac{2\pi x_2}{L} + \sin \frac{2\pi x_1}{L} \sin \frac{\pi x_2}{L} \right]$$

For fermions, the state must be antisymmetric, so we have

$$(6) \quad |x_1, x_2\rangle = \frac{\sqrt{2}}{L} \left[\sin \frac{\pi x_1}{L} \sin \frac{2\pi x_2}{L} - \sin \frac{2\pi x_1}{L} \sin \frac{\pi x_2}{L} \right]$$