FERMIONS AND BOSONS IN THE INFINITE SQUARE WELL

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Shankar, R. (1994), *Principles of Quantum Mechanics*, Plenum Press. Chapter 10, Exercise 10.3.4.

Suppose we have two identical particles in an infinite square well. The energy levels in a well of width L are

$$E=\frac{(\pi n\hbar)^2}{2mL^2} \tag{1}$$

where $n=1,2,3,\ldots$ The corresponding wave functions are given by

$$\psi_n(x)=\sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L} \tag{2}$$

If the total energy of the two particles is $\pi^2\hbar^2/mL^2$, the only possible configuration is for both particles to be in the ground state $n=1$. This means the particles must be bosons, so the state vector is

$$|x_1,x_2\rangle=\frac{2}{L}\sin\frac{\pi x_1}{L}\sin\frac{\pi x_2}{L} \tag{3}$$

If the total energy is $5\pi^2\hbar^2/2mL^2$, then one particle is in the state $n=1$ and the other is in $n=2$. Since the states are different, the particles can be either bosons or fermions. For bosons, the state vector is

$$\begin{aligned}
|x_1,x_2\rangle &= \frac{1}{\sqrt{2}}\left[\frac{2}{L}\sin\frac{\pi x_1}{L}\sin\frac{2\pi x_2}{L}+\frac{2}{L}\sin\frac{2\pi x_1}{L}\sin\frac{\pi x_2}{L}\right] && (4)\\
&= \frac{\sqrt{2}}{L}\left[\sin\frac{\pi x_1}{L}\sin\frac{2\pi x_2}{L}+\sin\frac{2\pi x_1}{L}\sin\frac{\pi x_2}{L}\right] && (5)
\end{aligned}$$

For fermions, the state must be antisymmetric, so we have

$$|x_1,x_2\rangle=\frac{\sqrt{2}}{L}\left[\sin\frac{\pi x_1}{L}\sin\frac{2\pi x_2}{L}-\sin\frac{2\pi x_1}{L}\sin\frac{\pi x_2}{L}\right] \tag{6}$$