HERMITIAN OPERATORS - A FEW EXAMPLES

Here are a few more results about hermitian operators. Suppose we are given two hermitian operators $\Omega$ and $\Lambda$. We’ll look at some combinations of these operators.

The operator $\Omega \Lambda$ has the hermitian conjugate

$$ (\Omega \Lambda)^\dagger = \Lambda^\dagger \Omega^\dagger = \Lambda \Omega $$

Thus the product operator $\Omega \Lambda$ is hermitian only if $\Lambda$ and $\Omega$ commute.

The operator $\Omega \Lambda + \lambda \Omega$ for some complex scalar $\lambda$ has the hermitian conjugate

$$ (\Omega \Lambda + \lambda \Omega)^\dagger = \Lambda^\dagger \Omega^\dagger + \lambda^* \Omega^\dagger $$

$$ = \Lambda \Omega + \lambda^* \Omega $$

This operator is therefore hermitian only if $\Lambda$ and $\Omega$ commute and $\lambda$ is real.

The commutator has the hermitian conjugate

$$ [\Omega, \Lambda]^\dagger = (\Omega \Lambda - \Lambda \Omega)^\dagger $$

$$ = \Lambda \Omega - \Omega \Lambda $$

$$ = [\Lambda, \Omega] $$

$$ = - [\Omega, \Lambda] $$

Thus the commutator is anti-hermitian (the hermitian conjugate is the negative of the original operator).

Finally, what happens if we multiply the commutator by $i$?
\[(i[\Omega, \Lambda])^\dagger = -i(\Omega \Lambda - \Lambda \Omega)^\dagger\] (8)
\[= -i(\Lambda \Omega - \Omega \Lambda)\] (9)
\[= -i[\Lambda, \Omega]\] (10)
\[= i[\Omega, \Lambda]\] (11)

Thus the operator \(i[\Omega, \Lambda]\) is hermitian.